Parabolic Riemann surfaces and Green Griffiths conjecture

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A conjecture by Green-Griffiths states that if X is a projective manifold of general type, then there exists an algebraic proper subvariety of X which contains the image of all holomorphic curves from the complex plane to X. The conjecture is far from being settled. I will question the choice of the complex plane as a source space.

Let Y be a parabolic Riemann surface, i.e. bounded subharmonic functions defined on Y are constant. The results of Nevanlinna's theory for holomorphic maps f from Y to the projective line are parallel to the classical case when Y is the complex line except for a term involving a weighted Euler characteristic. Parabolic Riemann Surfaces could be Hyperbolic in the Kobayashi sense.

Let X be a manifold of general type, and let A be an ample line bundle on X. It is known that there exists a holomorphic jet differential P (of order k) with values in the dual of A. If the map f has infinite area and if Y has finite Euler characteristic, then f satisfies the differential relation induced by P.

As a consequence, we obtain a generalization of Bloch Theorem concerning the Zariski closure of maps f with values in a complex torus.

We then study the degree of Nevanlinna's current $T[f]$ associated to a parabolic leaf of a foliation F by Riemann surfaces on a compact complex manifold. We show that the degree of $T[f]$ on the tangent bundle of the foliation is bounded from below in terms of the counting function of f with respect to the singularities of F, and the Euler characteristic of Y. In the case of complex surfaces of general type, we obtain a complete analogue of McQuillan's result: a parabolic curve of infinite area and finite Euler characteristic tangent to F is not Zariski dense. That requires some analysis of the dynamics of foliations by Riemann Surfaces.

This is joint work with Mihai Paun.