

Applications of Hodge theory to Teichmüller dynamics

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The moduli space of Riemann surfaces with a holomorphic one-form carries an action of the group $SL(2, \mathbb{R})$. It is a generalization of the action on the unit tangent bundle of the modular surface and can be viewed as a “complexified” geodesic flow for the Teichmüller metric. The dynamics is similar to the case of homogeneous spaces, but also uses tools from algebraic geometry and variations of Hodge structure. After providing an introduction to this topic, I will discuss some recent results about orbit closures for this action. The works of Eskin, Mirzakhani, and Mohammadi describe their local structure as smooth orbifolds. I will explain why they turn out to be algebraic varieties, characterized by a mixture of algebro-geometric conditions (real multiplication and torsion on Jacobians).