Closed symmetric differentials on projective surfaces

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Symmetric differential s of degree m on a projective surface S is a holomoprhic section of $S^m\Omega^1$. In case of a surface symmetric differentials define a holomorphic web. Indeed on a two-dimensional tangent space $\mathrm{T}_x(S)$ symmetric differential is a homogeneous polynomial of degree m and hence defines (in general) m tangent directions which locally define m foliations F_i, $i = 1, \dots, m$. At a point where all local foliations are different and nonsingular we can represent $s = f dz_1 \dots dz_m$ where $z_i = constant$ locally defines the levels of the foliations F_i. The functions z_i are nonunique and s is called closed if we can find z_i so that f is constant. This defintion extends the well known notion of a closed differential form. There is well known relation between closed 1-differentials and topological properties of the surface. In my talk I will discuss similar results and conjectures for closed m-differentials on surfaces.